

NPSOR-92-006

**NAVAL POSTGRADUATE SCHOOL**  
Monterey, California

**AD-A247 068**



**DTIC**  
**ELECTE**  
**MAR 11 1992**  
**S B D**

**MODELING THE TEMPORAL RELATIONSHIP OF  
CASUALTY REPORTS TO THE OPERATIONAL  
PROPULSION PLANT EXAM**

Lyn R. Whitaker  
and  
Robert R. Read

December 1991

Approved for public release; distribution is unlimited.

Prepared for:  
Dean of Instruction  
Naval Postgraduate School,  
Monterey, CA

**92-06188**



92 06 188

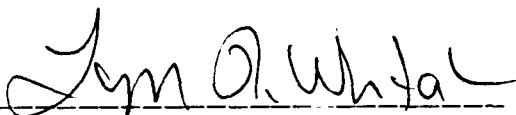
**NAVAL POSTGRADUATE SCHOOL,  
MONTEREY, CALIFORNIA**

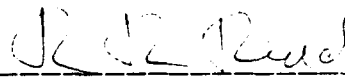
Rear Admiral R. W. West, Jr.  
Superintendent

Harrison Shull  
Provost

This report was funded by the Dean of Instruction, Naval Postgraduate School, Monterey, CA.


This report was prepared by:


  
\_\_\_\_\_  
LYN R. WHITAKER  
Assistant Professor,  
Department of Operations Research

  
\_\_\_\_\_  
ROBERT R. READ  
Professor,  
Department of Operations Research

Reviewed by:

Released by:

  
\_\_\_\_\_  
PETER PURDUE  
Professor and Chairman  
Department of Operations Research

  
\_\_\_\_\_  
PAUL J. MARTO  
Dean of Research

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

## REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

1. REPORT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>		1b. RESTRICTIVE MARKINGS	
2. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION /AVAILABILITY OF REPORT <b>Approved for public release; distribution is unlimited.</b>	
4. DECLASSIFICATION/DOWNGRADING SCHEDULE		5. MONITORING ORGANIZATION REPORT NUMBER(S)	
6. PERFORMING ORGANIZATION REPORT NUMBER(S) <b>NPSOR-92-006</b>		7a. NAME OF MONITORING ORGANIZATION <b>Naval Postgraduate School</b>	
7b. ADDRESS (City, State, and ZIP Code) <b>Monterey, CA 93943</b>	8a. NAME OF FUNDING/SPONSORING ORGANIZATION <b>Dean of Instruction</b>	8b. OFFICE SYMBOL (If applicable) <b>OR</b>	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER <b>OM&amp;N Direct Funding</b>
10. SOURCE OF FUNDING NUMBERS	11. TITLE (Include Security Classification) <b>Modeling the Temporal Relationship of Casualty Reports to the Operational Propulsion Plant Exam</b>		
12. PERSONAL AUTHOR(S) <b>Robert R. Read and Lyn R. Whitaker</b>	13. TYPE OF REPORT <b>Technical</b>		
14. DATE OF REPORT (Year, month day) <b>December, 1991</b>	15. PAGE COUNT		
16. SUPPLEMENTARY NOTATION			
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	<b>Categorical data analysis; loglinear models; Operational Propulsion Plant Exam</b>
19. ABSTRACT (Continue on reverse if necessary and identify by block number)  This report applies modern categorical data analysis to the problem of describing the probability laws of casualty reports of United States ships of the line in relation to the type of casualty and temporal nearness of the Operational Propulsion Plant Exam. It sets an example as to how data of this type are analyzed, to treat questions relating to competing modes of analysis, and to provide direction in the use of currently available software.			
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS		21. ABSTRACT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>	
22a. NAME OF RESPONSIBLE INDIVIDUAL <b>Whitaker</b>		22b. TELEPHONE (Include Area Code) <b>(408) 646-3482</b>	2c. OFFICE SYMBOL <b>OR/Wh</b>

D Form 1473, JUN 86

Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

S/N 0102-LF-014-6603

UNCLASSIFIED

# MODELING THE TEMPORAL RELATIONSHIP OF CASUALTY REPORTS TO THE OPERATIONAL PROPULSION PLANT EXAM

Robert R. Read and Lyn R. Whitaker\*

## Abstract

This report applies modern categorical data analysis to the problem of describing the probability laws of casualty reports of United States ships of the line in relation to the type of casualty and temporal nearness of the Operational Propulsion Plant Exam. It sets an example as to how data of this type are analyzed, to treat questions relating to competing modes of analysis, and to provide direction in the use of currently available software.

## 1. INTRODUCTION

This report applies modern categorical data analysis to the problem of describing the probability laws of the casualty reports (CASREPTs) of United States ships of the line in relation to the type of casualty (engineering or nonengineering) and the temporal nearness of the Operational Propulsion Plant Exam (OPPE). It is postulated that the preparation for the OPPE drains resources from normal maintenance operations in a way that induces an increase in the number of casualty reports as the time of the exam approaches. Also, the number of casualty reports diminishes monotonically with time in

---

\* This research was supported by course development funds of the Naval Postgraduate School, Monterey, California.

Availability Codes	
Dist	Avail and/or Special
A-1	

the post exam period as the system recovers from the effect. Further the effect may be different for engineering and nonengineering casualty reports.

The Navy created the Propulsion Examining Board (PEB) in 1972. It was tasked with inspecting the propulsion plants of the Navy's surface ships. The OPPE exam is first conducted approximately fifteen months after a ship has completed a regular overhaul, and is repeated about every fifteen months thereafter until the ship again enters the overhaul state. The PEB has the authority to "tie up" a ship which, in its opinion, has an engineering plant that is not safe to operate or does not have enough qualified engineering watch standers to operate it properly. Each fleet, Atlantic and Pacific, controls its own PEB and there may be differences in policies that affect the results.<sup>1</sup>

This study is restricted to frigates, destroyers and cruisers in each fleet possessing a 1200 PSI steam engineering plant. The time period is January 1974 to July 1978, and only those CASREPTs with a C-3 or C-4 readiness code are considered. The data are extracted from Tables 18 and 19 of the master's thesis of F. J. Klingseis (1979), who obtained them from CNA. The thesis mentions other data caveats as well. There are some discrepancies of these data from those of his Table 16. There is no way to resolve the discrepancies, since the data are old. Nonetheless we pursue the development. Our goal is to set an example as to how data of this type are to be analyzed, to treat questions relating to competing modes of analysis, and to provide direction

---

<sup>1</sup>Beginning early in 1992, the inspections for the two fleets will be made identical.

in the use of currently available software. Implementation of the methods for current use is left to others.

The raw data appear in Table 1. It may be viewed as being six dimensional with

$$x_{ijklrs} \quad (1)$$

representing the frequency count of exactly  $(r-1) = 0, 1, 2, 3$  or more CASREPTs in months  $(s = 1, \dots, 6)$  measured before  $(i = 1)$  or after  $(i = 2)$  the date of the OPPE; having been typed as engineering  $(k = 1)$  or nonengineering  $(k = 2)$ ; for ships of the class frigates, destroyers, or cruisers  $(l = 1, 2, 3 \text{ resp.})$ ; and belonging to the Atlantic  $(j = 1)$  or Pacific  $(j = 2)$  fleets. Thus there are 576 cells of counts. A visual inspection of Table 1 is not very revealing. The number of ships by fleet in each class is treated as fixed by design. Specifically

$$x_{ijkl+s} = N_{jl} \quad (2)$$

and these values appear in Table 2. It is important to note that we do not have information by individual ships, only the totals for ship class by fleet. This kind of collapsing is a bit unsettling as much detail is lost.

The first round of analyses are the elementary and naive ones. These treat the cells as 24 (before/after by two fleets by two casualty types by three ship classes) separate 4 by 6 (frequency categories by months) contingency tables. The basic *chi*-square test for common distribution over the months can be

TABLE 1. TWENTY-FOUR 4 × 6 FREQUENCY TABLES OF CASREPTS

ATLANTIC

PACIFIC

FRIGATES—BEFORE

Engineering						Nonengineering						Engineering						Nonengineering					
16	25	26	25	30	36	23	19	17	18	27	28	18	19	20	21	25	28	21	19	19	21	24	27
17	10	7	10	11	6	19	12	11	15	12	9	7	10	8	14	6	7	14	15	13	11	6	9
6	5	6	7	1	2	5	7	7	7	3	6	6	6	9	1	4	3	2	4	6	3	4	4
5	4	5	2	2	0	6	6	9	4	2	1	9	5	3	4	5	2	3	2	2	5	6	0

FRIGATES—AFTER

Engineering						Nonengineering						Engineering						Nonengineering					
20	27	28	34	27	29	21	24	22	21	23	27	20	17	20	20	25	23	19	17	23	16	22	25
16	12	10	5	14	6	9	11	11	13	12	7	11	13	13	8	10	9	11	11	6	13	6	12
3	5	3	2	3	7	11	5	4	5	6	5	7	3	3	9	2	5	7	9	9	6	6	2
5	0	3	3	0	2	3	4	7	5	3	5	2	7	4	3	3	3	3	3	2	5	6	1

DESTROYERS—BEFORE

Engineering						Nonengineering						Engineering						Nonengineering					
13	16	17	16	18	23	10	11	8	16	12	13	8	7	11	10	13	13	11	7	11	10	11	11
8	9	3	7	4	3	10	9	7	5	6	7	5	7	4	2	1	1	5	5	1	2	3	3
4	1	5	1	2	0	5	5	8	4	7	2	3	4	3	2	2	1	2	1	3	3	1	1
4	3	4	5	5	3	4	4	6	4	4	7	2	0	0	4	2	3	0	5	3	3	3	3

DESTROYERS—AFTER

Engineering						Nonengineering						Engineering						Nonengineering					
15	22	23	20	21	25	10	14	12	11	18	16	12	6	11	9	6	10	10	5	6	10	7	12
9	2	6	5	6	1	12	9	8	11	7	5	3	4	4	3	7	6	4	5	5	1	3	1
1	3	0	3	1	2	3	4	4	4	3	3	2	3	1	5	4	2	1	3	4	4	5	1
4	2	0	1	1	1	4	1	4	2	1	5	1	5	2	1	1	0	3	5	3	3	3	4

CRUISERS—BEFORE

Engineering						Nonengineering						Engineering						Nonengineering					
5	6	6	5	7	7	4	3	4	4	4	5	3	4	3	5	5	5	4	5	6	7	9	5
1	0	1	0	1	0	1	1	0	2	2	3	5	2	5	3	2	1	5	4	2	1	1	3
1	1	0	3	0	1	2	0	1	1	1	0	0	2	1	2	2	0	1	1	0	0	0	1
1	1	1	0	0	0	1	4	3	1	1	0	2	2	1	0	1	4	0	0	2	2	0	1

CRUISERS—AFTER

Engineering						Nonengineering						Engineering						Nonengineering					
4	4	7	4	7	6	2	3	2	3	2	4	8	8	7	8	7	6	4	3	3	6	4	6
3	3	1	3	1	2	4	1	2	2	2	2	0	2	2	2	0	2	3	4	5	1	3	4
0	1	0	1	0	0	1	2	1	1	2	0	2	0	0	0	3	1	2	2	2	1	1	0
1	0	0	0	0	0	1	2	3	2	2	2	0	0	1	0	0	1	1	1	0	2	2	0

accepted. It is also true that 24 separate loglinear models which treat the months as ordinal data provide equally acceptable fits to the data. Moreover, the latter model exhibits the monotone change in frequency of casualties as originally hypothesized. These studies are contained and discussed further in Section 2 following this introduction. The main body of the report appears in Section 3 where a modern loglinear model is selected to describe the entire six dimensional data set. It is shown there that any reasonable loglinear model must include the casualty count by month interaction term.

Section 4 contains another model building effort based upon a specialized collapsing of the original data set. It provides some rather interesting contrasts. It is used largely for a logit analysis of the engineering versus nonengineering CASREPTs. The results are summarized in Section 5. An annotated SAS code for the developments in Section 3 is presented in Appendix A. The details of fitting censored Poisson and Geometric distributions to the 24 separate frequency tables appear in Appendix B.

**TABLE 2. NUMBER OF SHIPS BY CLASS AND FLEET ( $N_{jl}$ )**

	FF	DD	CG
Atlantic	44	29	8
Pacific	40	18	10

## **2. ELEMENTARY ANALYSES: TWENTY-FOUR SEPARATE CASES**

The standard procedure for testing whether the six months have a common four point distribution can be found in any basic statistics text, e.g., Agresti (1990), and the test statistics have an asymptotic *chi-squared*

distribution with 15 degrees of freedom when the null hypotheses are true. Assuming independence of the 24 data sets, then if all null hypotheses are valid, the  $p$ -values of the tests form a random sample from a Uniform  $[0,1]$  distribution. This is a consequence of the probability integral transformation. Thus, a test for the simultaneous validity of all 24 null hypotheses may be executed using a Kolmogorov-Smirnov test for uniformity of the distribution of the  $p$ -values. Here,  $p$  stands for the empirical significance level (the probability of a result at least as extreme if  $H_0$  were true).

The 24 test statistics appear in Table 3 below and their significance numbers, in the form of  $1-p$ , follow in Table 4. They too would be uniformly distributed if all null hypotheses were true. They appear to be smeared evenly over the unit interval. The Kolmogorov-Smirnov statistic is, for  $\{p_j\}$  equal to the ordered values of  $p$ ,

$$D_n = \max |p_j - j/n| = .183 \quad \text{and} \quad \Pr\{\sqrt{n} D_n \geq .183\} = .401$$

and there is temptation to stop the analysis here.

TABLE 3. CHI-SQUARE VALUES FOR CASUALTY COUNTS  
INDEPENDENT OF MONTH

		ATLANTIC		PACIFIC	
		Engineering	Nonengineering	Engineering	Nonengineering
FF	b	28.84	20.17	22.75	17.60
	a	25.87	11.26	15.67	18.23
DD	b	19.12	11.81	21.03	12.48
	a	22.36	12.87	19.58	15.46
CG	b	12.67	14.37	15.57	16.47
	a	13.36	6.91	16.45	11.45

At this level one should realize that failing to reject a particular model does not preclude the acceptability of a competing model. Indeed, the power of the *chi-square* goodness-of-fit procedure is not great. Accordingly we try our luck with loglinear models that allow for variability of the casualty frequency distribution by month. Moreover month is to be treated as a scored ordinal variable. If an acceptable fit is achieved then we look for monotonicity of change by month.

TABLE 4. SIGNIFICANCE (1-*p* VALUES) OF TABLE 3 STATISTICS

		ATLANTIC		PACIFIC	
		Engineering	Nonengineering	Engineering	Nonengineering
FF	b	.983	.835	.910	.716
	a	.961	.266	.595	.749
DD	b	.792	.307	.864	.358
	a	.901	.388	.811	.581
CG	b	.372	.502	.589	.648
	a	.425	.040	.647	.280

It is interesting to note that the total number of ships constraint, see Table 2, has a profound effect upon the choice of model to be fitted. We begin with the simplest.

Since we are treating the 24 tables separately, we drop all subscripts except *r* and *s* for the time being. Let  $m_{rs}$  be the expected cell frequency; adopt the simple ordinal scoring  $v_s = s$  for  $s = 1, \dots, 6$  and  $\bar{v} = 3.5$ . Consider the loglinear model

$$\log(m_{rs}) = \mu + \delta_r(v_s - \bar{v}) \quad (3)$$

with  $\sum \delta_r = 0$ , and  $r$  ranges 1, ..., 4.

The total number of ships constraint requires that all

$$m_{+s} = N \quad \text{for } s = 1, \dots, 6 \quad (4)$$

where  $N$  is the appropriate number from Table 2. In terms of our model this requires

$$m_{+s} = e^{\mu} \sum_r e^{\delta_r (v_s - \bar{v})} \quad (5)$$

which can happen only if all  $\delta_r = 0$ . This in turn confiscates all usefulness of the model. The same analysis leads to the rejection of the model

$$\log(m_{rs}) = \mu + \alpha_r + \delta_r (v_s - \bar{v}). \quad (6)$$

The simplest feasible model with months taken to be ordinal is

$$\log(m_{rs}) = \mu + \alpha_r + \beta_s + \delta_r (v_s - \bar{v}) \quad (7)$$

with  $\sum \alpha_r = \sum \beta_s = \sum \delta_r = 0$ . The cells means are estimated by iterated proportional scaling.<sup>2</sup> The 24 *chi*-square goodness-of-fit test statistics appear in Table 5 (12 degrees of freedom) and their significance values in Table 6.

---

<sup>2</sup>Computational support is discussed in Sections 3 and 4. Note that PROC CATMOD of SAS Version 6.06 does not have a command to fit loglinear models with ordinal explanatory variables. However, such a model can be fit using PROC CATMOD by specifying the appropriate design matrix.

Again the Kolmogorov-Smirnov procedure is performed for the 24  $p$ -values, producing

$$D_n = \max |p_j - j/n| = .173 \quad \text{and} \quad \Pr\{\sqrt{n} D_n \geq .173\} = .472. \quad (8)$$

Thus both models fit the data equally well.

Let us examine our estimates of the probability to see if the number casualties estimated by the model decrease with time. Table 7 contains a compilation of the probability of zero casualty reports for each month for each of our 24 cases. In 22 of the cases the probabilities grow monotonically by month, thus supporting our assertion. It also seems that the probabilities for engineering casualties change more than those for nonengineering. It may be curious to note that in the odd cases, engineering casualties after OPPE for Pacific cruisers and destroyers, that the probabilities are strictly decreasing with month. On the other hand, the probabilities of casualties (average engineering and nonengineering), follow the asserted monotone increasing pattern. It is instructive for the reader to compare the cumulative distribution functions that result from fitting (7).

TABLE 5. GOODNESS-OF-FIT VALUES FOR THE LOGLINEAR MODEL

		ATLANTIC		PACIFIC	
		Engineering	Nonengineering	Engineering	Nonengineering
FF	b	10.68	8.88	9.22	11.44
	a	14.74	8.88	13.16	12.16
DD	b	10.55	8.55	11.61	14.00
	a	6.82	5.92	15.87	9.06
CG	b	12.40	13.17	12.62	18.83
	a	8.38	13.12	14.71	10.68

TABLE 6. SIGNIFICANCE (1- $p$  VALUES) OF TABLE 5 STATISTICS

		ATLANTIC		PACIFIC	
		Engineering	Nonengineering	Engineering	Nonengineering
FF	b	.443	.287	.316	.508
	a	.744	.287	.642	.575
DD	b	.432	.259	.522	.699
	a	.130	.080	.803	.302
CG	b	.582	.643	.603	.907
	a	.246	.639	.742	.443

TABLE 7. PROBABILITY OF ZERO CASREPTS LOGLINEAR MODEL

		Engineering							Nonengineering						
FF	b	Atlantic	.405	.487	.568	.645	.713	.772	.377	.422	.467	.511	.553	.594	
		Pacific	.418	.472	.525	.575	.621	.664	.459	.495	.532	.566	.597	.626	
	a	Atlantic	.536	.576	.613	.647	.676	.701	.479	.498	.516	.532	.548	.563	
		Pacific	.451	.479	.507	.535	.563	.590	.438	.468	.497	.524	.550	.574	
DD	b	Atlantic	.448	.512	.572	.627	.676	.718	.337	.364	.390	.416	.441	.465	
		Pacific	.381	.476	.563	.632	.681	.711	.517	.539	.560	.577	.592	.604	
	a	Atlantic	.604	.662	.712	.755	.791	.821	.374	.414	.454	.494	.533	.570	
		Pacific	.534	.532	.520	.500	.473	.441	.378	.416	.452	.483	.512	.537	
CG	b	Atlantic	.615	.695	.751	.789	.815	.835	.398	.461	.511	.542	.551	.538	
		Pacific	.304	.353	.400	.444	.483	.517	.458	.524	.585	.638	.681	.714	
	a	Atlantic	.485	.606	.659	.707	.751	.790	.262	.290	.319	.238	.376	.404	
		Pacific	.812	.786	.756	.722	.684	.640	.315	.365	.414	.460	.503	.543	

To conclude this section we note that, with a quick look at the  $p$ -values, both models are defensible and the one that models the casualty distribution as a function of time clearly supports our conjecture. We also know the power of the statistical procedure is not high. More importantly, the 24 cases are not independent. The cross classifications of before/after and engineering/ nonengineering refer to the same ships. There are but six (fleet by ship class) independent data sets.

### 3. A LOGLINEAR MODEL

The analysis in the previous section gives two separate acceptable models, one indicating that temporal nearness to the OPPE exam has no effect on the number of CASREPTs and a contradictory model indicating that temporal nearness does indeed have an effect. A closer look at the  $1-p$ -values from Tables 4 and 6 help clear up this discrepancy and motivate the need to consider the data as a whole. If the models fit, i.e. the  $p$ -values (or equivalently the  $1-p$ -values) form simple random samples from a Uniform  $[0,1]$  distribution, then subsets of the  $p$ -values should also behave as simple random samples from a Uniform  $[0,1]$  distribution. Figures 1 and 2 give box-plots of the  $1-p$ -values from Tables 6 and 8 respectively by ship type and by casualty type. In Figure 1, there is clearly some effect that the first set of models is not picking up. This effect (Figure 2) is given considerable relief when temporal nearness is included in the models. We note that the Kolmogorov-Smirnov procedure does not have power to detect all types of departures from the null hypothesis. In particular it cannot detect patterns such as those exhibited in Figure 1. (The independence assumption is important.) From these figures we can conclude that temporal nearness is indeed a variable that needs to be considered, and that there is interaction between temporal nearness and the other variables.

In this section we treat the data as a whole, using a loglinear model in order to get a better idea of the interaction between temporal nearness and the other variables and their effect on the number of CASREPTs. The first step is to choose the main effects and interaction terms for inclusion. There are

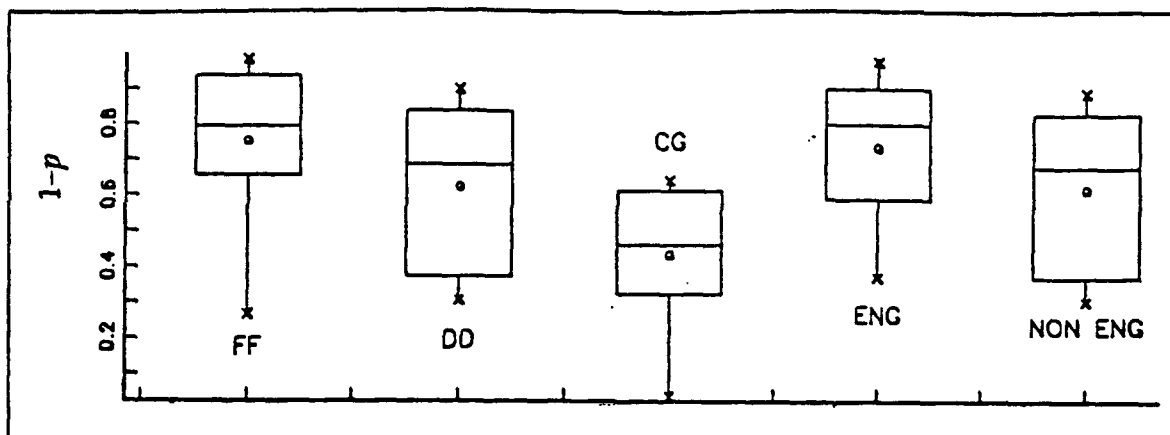


Figure 1. Box Plots of  $1-p$ -values from Table 5 by Ship Type and by Casualty Type

several strategies, similar to model selection in regression settings, for doing this (e.g., Agresti (1990)). Our strategy is motivated by the available software as well as by certain aspects of the problem. Thus, an important feature of this section is the computational difficulties and methods for solving them.

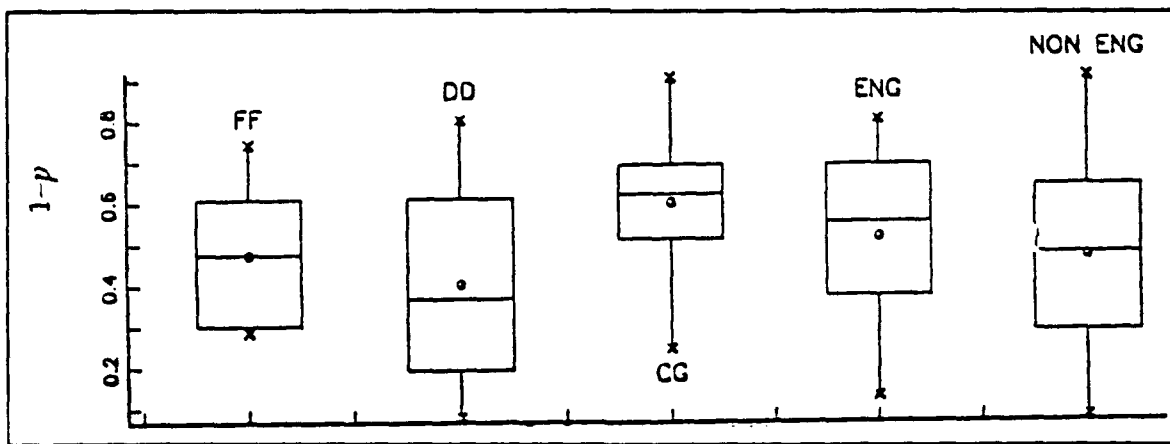


Figure 2. Box Plots of  $1-p$ -values from Table 6 by Ship Type and by Casualty Type

The cell counts are not realizations from a single multinomial distribution since the number of ships by fleet is fixed, and the number of casualties is reported for the same ships for both casualty types over the 12-month period

surrounding the OPPE. They can be modeled as realizations from several multinomial distributions. In particular, for each  $i, j, k, l, s$  the random variables of the form

$$X_{ijkl0s}, \dots, X_{ijkl3s} \mid X_{ijkl+s} = N_{il} \quad (9)$$

have multinomial distributions where  $X_{ijklrs}$  is the random variable corresponding to the observed frequency  $x_{ijklrs}$ . The natural inclination is to take the likelihood to be the product of these multinomials and continue from there. By doing this we are tacitly assuming that the number of casualties per casualty type and month before and after OPPE are independent within each ship type by fleet as well as between ship types and fleets. The disturbing part of this assumption is that for each ship type by fleet, the same ships are observed over the 12-month period surrounding the OPPE. If we had data by ship, it might be possible to take into account potential dependence in the number of casualties within a ship using a repeated measures design. However, we don't have the data.

Some statistical packages such as SAS are able to maximize products of multinomials, others are not. Birch (see Agresti (1990:p. 169)) showed that the MLEs for a multinomial likelihood are the same as the MLEs for product multinomials as long as the model contains a term for the marginal distribution fixed by the sampling design. In this problem the number of counts of ship by fleet by casualty type by month by before and after OPPE is fixed. Thus designating the design factors and levels as follows

Factor	No. Levels	Levels
A	2	Before/after OPPE $i = 1, 2$ ,
B	2	Atlantic, Pacific fleets, $j = 1, 2$

C	2	Engineering, nonengineering casualty types, $k = 1, 2$
D	3	Frigates, Destroyers, Cruisers, $l = 1, 2, 3$
E	6	months measured from the time of OPPE $s = 1, \dots, 6$
F	4	0, 1, 2, 3 or more CASREPTs, $r = 1, 2, 3, 4$

we can use a package that does not explicitly maximize the product of independent multinomials by including the 5-way interaction term ABCDE.

The goal is to find a reasonable model that fits the data, but does not include too many parameters. This is an iterative process somewhat similar to stepwise regression. We begin by fitting the model with all main effects and ABCDE (likelihood ratio = 663.67 with 562 degrees of freedom and a  $p$ -value = .0019), and the model with main effects, all two-way interactions and ABCDE (likelihood ratio = 496.52, with 488 degrees of freedom and a  $p$ -value = .3850). The model with all two-way interactions appears to fit the data. Thus, we use this model as a starting point and then eliminate parameters sequentially until we get a model that is no longer suitable. Backwards elimination is much easier and safer than forward selection if you don't have a computer package that does some type of model selection. Which terms to eliminate can be decided by looking at the output from one run of the more expansive model. Forward selection requires that a new model be fit for each term that you might want to add to the model. Starting from the model with just main effects, we would need to make 15 runs to decide which of the two-way interaction terms produces the greatest improvement. SAS version 6.06 was used, even though it does not have a stepwise model selection option, because it allows the inclusion of higher order interaction terms such as ABCDE, without requiring that all lower order terms be present.

We remove the terms with the highest  $p$ -value for the test of the null hypothesis that the terms are insignificant. From the model with all two-way

interaction terms (see Table 8) we remove AB, AC, AD, AE, BC, BE, CD, CE and DE. All have  $p$ -values  $> .8$ . Note that ABCDE is retained despite the fact that it has a  $p$ -value = 1.0000. Even though including this term does not affect the estimates of the other parameters, or the test statistics, it is needed to provide the correct degrees of freedom for the model, 488 versus 498. Changing the degrees of freedom from 488 to 498 alters the  $p$ -value for the model rather drastically from 0.3850 to 0.6. After removing these terms we have the model output given in Table 9.

The overall likelihood ratio test statistic changes slightly from 496.52 with 488 degrees of freedom to 497.19 with 520 degrees of freedom. This difference 0.67 with 12 degrees of freedom indicates that there is no real difference in the fits of these two models. When eliminating more than one term it is important to check the difference in the model fits. It could happen that in the presence of all the other terms each term by itself is insignificant, but, that taken together with the resulting model does not fit. This is exactly what happens were we to remove all terms, (except ABCDE) with  $p$ -values  $> 0.3$ .

It is clear from the  $p$ -values in Table 9 that we are close to a final model, thus we now remove terms one at a time. First A, then BF, then AF (see Tables 9-11) to get the model in Table 12. In Table 10 the  $p$ -value for B in the presence of BF is 0.1195. However, once BF is eliminated, see Table 11, the  $p$ -value for B is 0.0045 indicating that both B and BF are explaining the same variability in the cell frequencies, and that it would have been a mistake to remove both of them. No further terms can be eliminated from Table 12 without significantly changing the model fit. It is interesting to note that eliminating factor A (before and after) has the effect of combining cells, i.e. eliminates the subscript  $i$ .

**TABLE 8. ANALYSIS OF VARIANCE TABLE FOR THE MODEL WITH ALL MAIN EFFECTS, ALL TWO-WAY INTERACTION TERMS, AND THE ABCDE TERM**

Source	Degrees of Freedom	Chi-square	p-value
A (before and after OPPE)	1	0.31	0.5783
AB	1	0.00	0.9879*
AC	1	0.01	0.9384*
AD	2	0.02	0.9911*
AE	5	0.00	1.0000*
AF	3	5.52	0.1376
B (Fleet)	1	2.35	0.1252
BC	1	0.03	0.8603*
BD	2	44.94	0.0000
BE	5	0.04	1.0000*
BF	3	3.32	0.3447
C (Casualty type)	1	8.71	0.0032
CD	2	0.02	0.9881*
CE	5	0.54	0.9905*
CF	3	37.11	0.0000
D (Ship type)	2	564.98	0.0000
DE	10	0.02	1.0000*
CF	6	14.99	0.0203
E (Month)	5	10.87	0.0541
EF	15	60.85	0.0000
F (CASREPTs)	3	1022.61	0.0000
ABCDE	10	0.00	1.0000
Likelihood Ratio	488	496.52	0.3850

**TABLE 9. ANALYSIS OF VARIANCE TABLE FOR THE MODEL OF TABLE 8 EXCLUDING THE ASTERISKED TERMS IN TABLE 8**

Source	Degrees of Freedom	Chi-square	p-value
A (before and after OPPE)	1	0.45	0.5012*
AF	3	5.49	0.1391
B (Fleet)	1	2.42	0.1195
BD	2	44.93	0.0000
BF	3	3.25	0.3546
C (Casualty type)	1	11.14	0.0008
CF	3	36.53	0.0000
D (Ship type)	2	569.00	0.0000
CF	6	14.94	0.0208
E (Month)	5	13.39	0.0200
EF	15	60.28	0.0000
F (CASREPTs)	3	1030.18	0.0000
ABCDE	10	0.00	1.0000
Likelihood Ratio	520	497.19	0.7572

**TABLE 10. ANALYSIS OF VARIANCE TABLE FOR THE MODEL IN  
TABLE 9 EXCLUDING THE AF TERM**

<b>Source</b>	<b>Degrees of Freedom</b>	<b>Chi-square</b>	<b>p-value</b>
AF	3	5.06	0.1677
B (Fleet)	1	2.42	0.1195
BD	2	44.93	0.0000
BF	3	3.25	0.3546*
C (Casualty type)	1	11.14	0.0008
CF	3	36.53	0.0000
D (Ship type)	2	569.01	0.0000
DF	6	14.94	0.0208
E (Month)	5	13.38	0.0200
EF	15	60.28	0.0000
F (CASREPTs)	3	1030.29	0.0000
ABCDE	10	0.00	1.0000
Likelihood Ratio	521	497.64	0.7624

**TABLE 11. ANALYSIS OF VARIANCE TABLE FOR THE MODEL IN  
TABLE 10 EXCLUDING THE BF TERM**

<b>Source</b>	<b>Degrees of Freedom</b>	<b>Chi-square</b>	<b>p-value</b>
AF	3	5.06	0.1677*
B (Fleet)	1	8.06	0.0045
BD	2	44.40	0.0000
C (Casualty type)	1	11.14	0.0008
CF	3	36.53	0.0000
D (Ship type)	2	568.85	0.0000
DF	6	14.39	0.0256
E (Month)	5	13.39	0.0200
EF	15	60.28	0.0000
F (CASREPTs)	3	1036.71	0.0000
ABCDE	10	0.00	1.0000
Likelihood Ratio	524	500.89	0.7593

TABLE 12. ANALYSIS OF VARIANCE TABLE FOR THE FINAL MODEL

Source	Degrees of Freedom	Chi-square	p-value
B (Fleet)	1	8.06	0.0045
BD	2	44.40	0.0000
C (Casualty type)	1	11.14	0.0008
CF	3	36.53	0.0000
D (Ship type)	2	568.85	0.0000
DF	6	14.38	0.0256
E (Month)	5	13.39	0.0200
EF	15	60.28	0.0000
F (CASREPTs)	3	1035.71	0.0000
ABCDE	10	0.00	1.0000
Likelihood Ratio	527	505.96	0.7377

The final loglinear model is

$$\ln p_{ijklrs} = \alpha + \alpha_j^B + \alpha_k^C + \alpha_l^D + \alpha_s^E + \alpha_r^F + \alpha_{jl}^{BD} + \alpha_{kr}^{CF} + \alpha_{lr}^{DF} + \alpha_{sr}^{EF} + \alpha_{ijkl}^{ABCDE},$$

with the appropriate constraints on the parameters, and where  $p_{ijklrs}$  would represent the probability of an observation falling into cell  $ijklrs$  had we been sampling from a single multinomial distribution. In our case, the parameters of the 72 (because factor A is eliminated) individual multinomial distributions, i.e. the distributions for the number of CASREPTs (0, 1, 2, 3 or more) everything else ( $ijkl$ s) being fixed, are

$$p_{r|ijkl} = \frac{p_{ijklrs}}{\sum_r p_{ijklrs}} \quad \text{for } r = 1, \dots, 4.$$

Since all terms not involving  $F$  cancel, the estimates of these probabilities are

$$\hat{p}_r|ijkl s = \frac{\exp\{\hat{\alpha}_r^F + \hat{\alpha}_{kr}^{CF} + \hat{\alpha}_{lr}^{DF} + \hat{\alpha}_{sr}^{EF}\}}{\sum_r \exp\{\hat{\alpha}_r^F + \hat{\alpha}_{kr}^{CF} + \hat{\alpha}_{lr}^{DF} + \hat{\alpha}_{sr}^{EF}\}},$$

and are given in Table 13, as percentages. This tells us that given casualty type, ship type and month that fleet has no effect on the distribution of the number of CASREPTs. Because the final loglinear model has a BD interaction term, it appears that differences in the fleets are due to the fact that the fleets have a different mix of shiptypes (see Table 2). For each ship type by casualty type by fleet, the estimated probabilities of no CASREPTs in a given month are increasing with distance from the OPPE exam. The same cannot be said for the estimated probabilities of three or more CASREPTs; these increase then decrease with nearness to the OPPE exam. But this is due mostly to the fact that probability functions must sum to one. When cumulative distributions are compared, the monotonicity by month is (essentially) supported. Across all ship types and months the estimated distribution for Nonengineering CASREPTs is stochastically greater than for Engineering CASREPTs.

The difference in the distributions of CASREPTs between ship types is not so clearcut; frigates tend to have the fewest CASREPTs followed by destroyers then cruisers. In this model, either casualty type or ship type interact with the number of CASREPTs by month.

Fitting this type of loglinear model is not the only way to analyze this data. In the next section a substantially different approach is used which uncovers structure in the data not apparent from the analysis in this section.

**TABLE 13. ESTIMATES OF THE DISTRIBUTIONS OF THE NUMBER OF CASREPTS BY MONTH, CASUALTY TYPE AND SHIP TYPE**

**Frigates, Engineering**

	1	2	3	4	5	6
0	52.06	53.84	57.57	58.72	64.17	69.89
1	28.63	26.61	22.25	22.79	20.40	17.45
2	11.42	11.55	11.97	11.16	9.14	7.03
≥3	7.88	7.99	8.21	7.32	6.29	5.62

**Frigates, Nonengineering**

	1	2	3	4	5	6
0	42.54	44.21	47.77	49.06	54.86	61.18
1	32.10	30.07	25.41	26.20	24.01	21.02
2	14.56	14.80	15.49	14.55	12.19	9.60
≥3	10.71	10.91	11.53	10.18	8.94	8.20

**Destroyers, Engineering**

	1	2	3	4	5	6
0	51.66	53.27	56.62	58.03	63.59	69.30
1	25.22	23.37	19.43	19.99	17.95	15.36
2	11.87	11.97	12.32	11.55	9.49	7.30
≥3	11.26	11.38	11.63	10.42	8.97	8.04

**Destroyers, Nonengineering**

	1	2	3	4	5	6
0	41.79	43.29	46.44	48.00	53.88	60.16
1	28.08	26.14	21.93	22.75	20.93	18.35
2	14.98	15.18	15.77	14.91	12.54	9.89
≥3	15.15	15.39	15.86	14.34	12.64	11.60

**Cruisers, Engineering**

	1	2	3	4	5	6
0	52.93	54.64	58.23	59.50	64.90	70.43
1	26.90	24.96	20.80	21.34	19.07	16.25
2	9.77	9.87	10.19	9.52	7.78	5.96
≥3	10.40	10.53	10.78	9.64	8.25	7.36

**Cruisers, Nonengineering**

	1	2	3	4	5	6
0	43.21	44.82	48.25	49.69	55.46	61.59
1	30.22	28.17	23.72	24.52	22.43	19.56
2	12.45	12.63	13.17	12.41	10.38	8.14
≥3	14.13	14.37	14.86	13.39	11.74	10.71

#### 4. COMPARISON OF ENGINEERING AND NONENGINEERING CASUALTY REPORTS

It is of interest to study the effects of the various factors upon the ratio of engineering and nonengineering CASREPTs. The concern is that resources may be diverted from nonengineering to engineering in order to prepare for the OPPE. Also there may be a postexam recovery effect. The particular technique chosen does not utilize the model developed in Section 3, but represents an alternative form of analysis. It is instructive to explore this alternative.

It begins with an attempt to simplify the data set by collapsing six dimensions to five. Specifically, let

$$Y_{ijkl s} = \sum_{r=1}^4 (r-1) X_{ijkdrs}$$

be the number of CASREPTs (more specifically a lower bound for the number) recorded in before/after category  $i$ , fleet  $j$ , casualty type  $k$ , ship class  $l$  in month  $s$ ; ( $i = 1, 2$ ;  $j = 1, 2$ ;  $k = 1, 2$ ;  $l = 1, 2, 3$ ;  $s = 1, 2, \dots, 6$ ).

These values have the advantage of containing no zeroes, having five dimensions vice six, and not possessing any restricting marginal totals such as those of Table 2. Thus, one might expect the data in this form to be simpler to model. We shall see however that it is in fact more difficult to model. The reason for this is that we do not have CASREPT information for the individual ships; we only have data for the cross-classification of fleet by ship class. In the cross-classified data, there are more CASREPTs for Atlantic frigates than Atlantic destroyers because there are more frigates than

destroyers in the Atlantic, etc. Many of the model effects estimated from this data structure are devoted to representing this information.

An additional reason for collapsing the data is to gain experience in the use of a second software system, specifically the categorical data analysis portion of STATGRAPHICS by STSC. This is an interactive package that can be used on PCs, features stepwise selection (both forward and backward) modeling, and allows graphical study of the residuals. On the negative side, this system treats only hierarchical models. If a certain interaction appears in the set of generators then all main effects and lower order interactions that can be constructed from the given generator must also appear in the model. Thus it is not possible to include an isolated high order interaction term for the purpose of treating a design constraint, as was done in Section 3. The factors and levels are designated as in Section 3.

It is instructive to relate some experiences in the artwork of modeling: The TEST ORDER option leads one to explore models containing 3-way effects. This done, the use of BACKWARD SELECTION is exploited to produce models that fit adequately and are parsimonious in terms of the number of effects included. This leads to the consideration of the model having generators

ABD ACD BCE BDE.

The fitting information for this set is

	Value	d.f.	p
Likelihood Ratio <i>chi</i> -square	91.0713	85	.3056
Pearson <i>chi</i> -square	85.4251	85	.4667

This model fits the data reasonably well and was chosen for further study to look for potential outliers and patterned residuals. Use of the STATGRAF plotting options on the standardized residuals reveal two outliers: (i) a value of -2.33 for Pacific cruisers, nonengineering, 5 months before the OPPE, and (ii) a value of 3.015 for Atlantic frigates, engineering, 6 months after the OPPE. An effort was made to improve the model by adding interaction terms even though these outliers were not especially severe. Also, the normal probability plot of residuals pointed to the possibility of improvement.

Accordingly, some additional exploration was performed and it was decided to include the ACE interaction term in the generators. This term alone costs 5 degrees of freedom and, because of the hierarchical nature of the algorithm, an additional 5 degrees of freedom are added for the AE interaction that must be included. Thus the finalized set of generators is

ABD   ACD   ACE   BCE   BDE

and the fitting summary is

	Value	d.f.	p
Likelihood Ratio <i>chi</i> -square	79.2540	75	.3463
Pearson <i>chi</i> -square	73.1462	75	.5391

The full loglinear model is

$$\begin{aligned}\ln(m_{ijkl s}) = & \lambda + \lambda_i^A + \lambda_j^B + \lambda_k^C + \lambda_l^D + \lambda_s^E \\ & + \lambda_{ij}^{AB} + \lambda_{ih}^{AC} + \lambda_{il}^{AD} + \lambda_{is}^{AE} + \lambda_{jh}^{BC} + \lambda_{il}^{BD} + \lambda_{js}^{BE} + \lambda_{kl}^{CD} + \lambda_{hs}^{CE} \\ & + \lambda_{ijl}^{ABD} + \lambda_{ihl}^{ACD} + \lambda_{ihs}^{ACE} + \lambda_{ihs}^{BCE} + \lambda_{jls}^{BDE}\end{aligned}$$

where  $m_{ijkl s} = E[Y_{ijkl s}]$ , for  $i = 1, 2; j = 1, 2; k = 1, 2; l = 1, 2; s = 1, \dots, 6$ , and the usual caveats for effects and interactions summing to zero. Plots of standardized residuals versus fitted values and Normal probability plots appear in Figure 3.

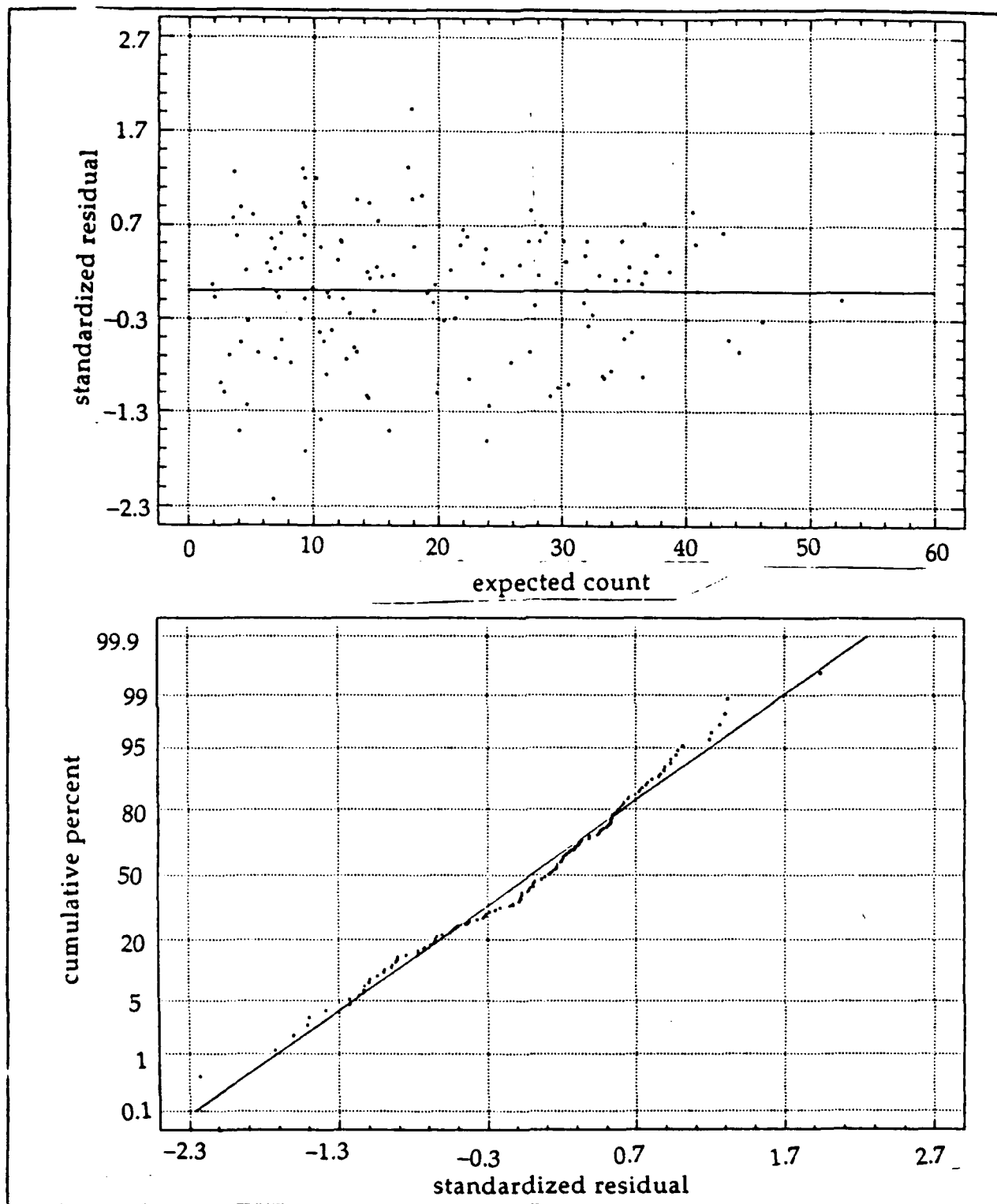
This fitted model will be used to study the behavior of log odds of engineering and nonengineering casualties. The induced model is

$$\ln\left(\frac{m_{ij1ls}}{m_{ij2ls}}\right) = \mu + \alpha_i + \beta_j + \gamma_{il} + \delta_{is} + \theta_{js}$$

which is not too overbearing. We must include the constraints  $\sum_i \alpha_i = \sum_j \beta_j$

$$= \sum_i \gamma_{il} = \sum_l \gamma_{il} = \sum_i \delta_{is} = \sum_s \delta_{is} = \sum_j \theta_{js} = \sum_s \theta_{js} = 0 \text{ for all } i, j, l, s. \text{ Positive values}$$

for the log odds mark engineering CASREPS as being favored (more prominent) and negative values favor the nonengineering type. Of course this represents a filtering of information, and the original model cannot be recovered from it.



**Figure 3. Plots of the Standardized Residuals vs. the Expected Counts and the Normal Probability Plots for the Final Model**

The effects are readily identified as

$$\mu = 2\lambda_1^C \quad \alpha_i = 2\lambda_{i1}^{AC} \quad \beta_j = 2\lambda_{j1}^{BC} \quad \gamma_{il} = 2\lambda_{i1l}^{ACD} \quad \delta_{is} = 2\lambda_{i1s}^{ACE} \quad \theta_{js} = 2\lambda_{j1s}^{BCE}$$

The figures show the six-month time traces of the log odds for before and after crossed by the three ship classes; Figure 4 treats the Atlantic fleet data and Figure 5 treats the Pacific. For both fleets the traces are generally parallel and the post-OPPE curves are below the pre-OPPE curves.

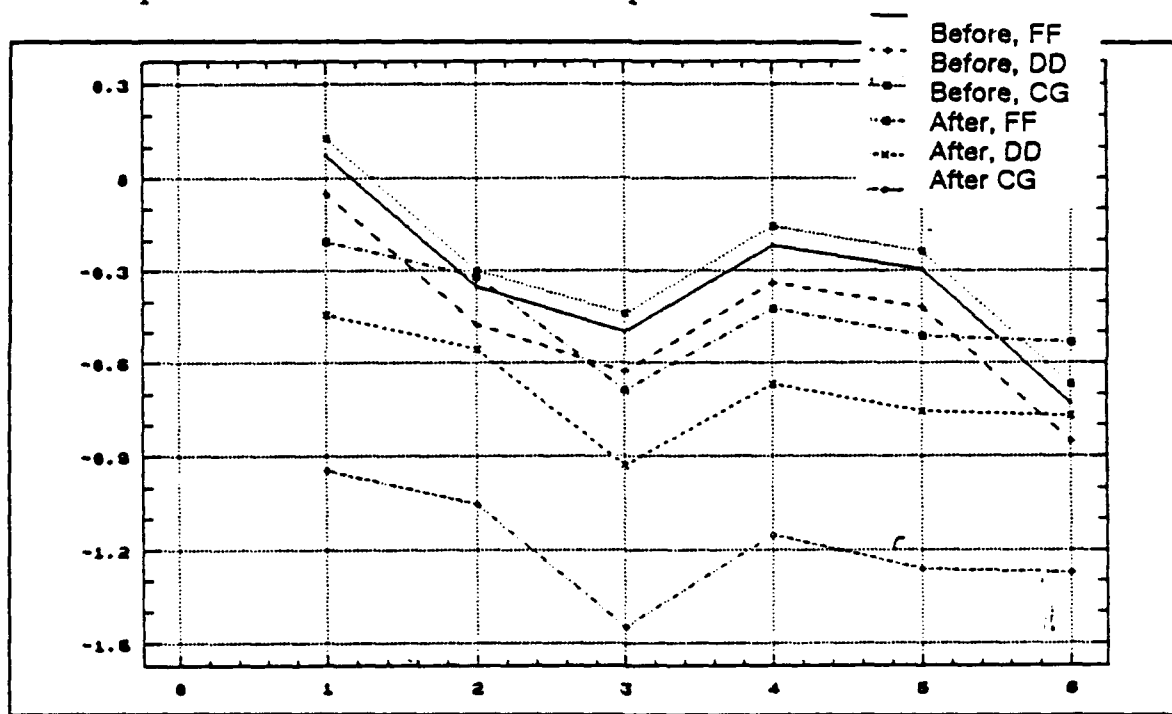


Figure 4. Traces of the Log Odds versus Month from OPPE for the Atlantic Fleet by Before/After and Ship Type

Thus, the transfer of resources effect might be associated with an imbalance of CASREPTs 2 to 5 months after OPPE for the Atlantic fleet; and 1 to 4 months after for the Pacific fleet. For both fleets, the curves for cruisers are sharply separated in the before and after effect. The curves for the

destroyers show a bit less separation, and those for frigates even less. In fact the pre- and post-frigate curves actually intersect.

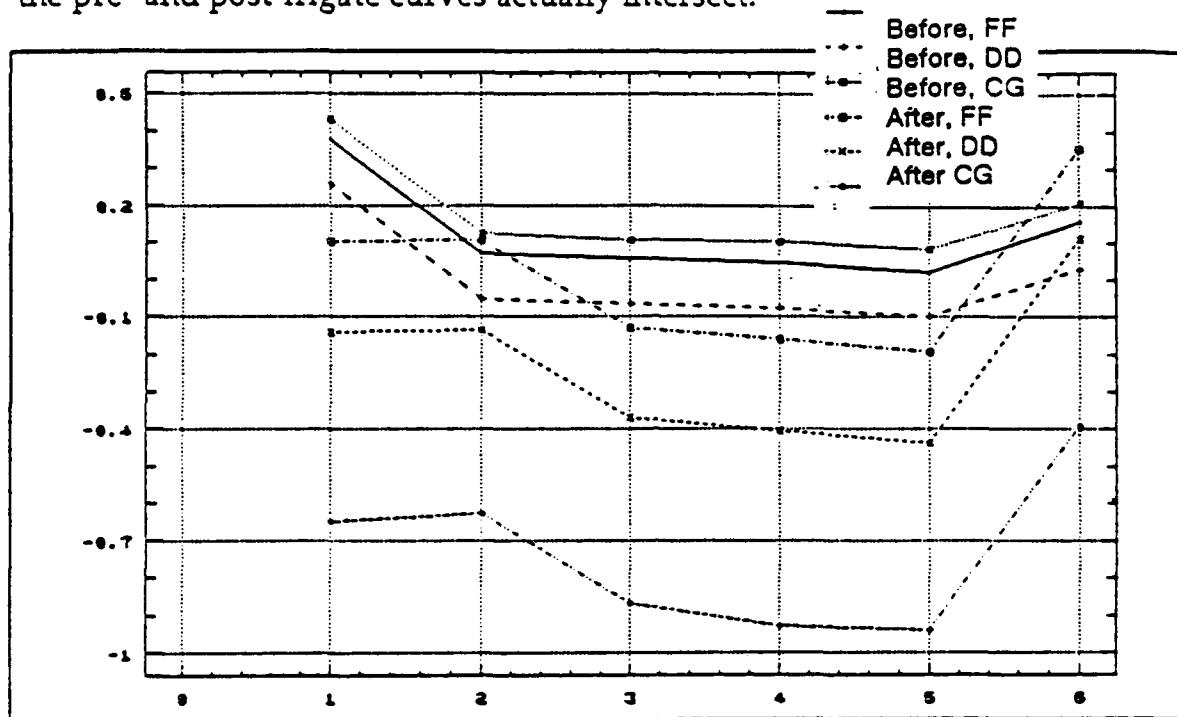


Figure 5. Traces of the Log Odds versus Month from OPPE for the Pacific Fleet by Before/After and Ship Type

These results are not inconsistent with those of Section 3. An estimate of a lower bound for the expected number of CASREPTs can be found from Table 13 by

$$\sum_{r=1}^4 (r-1) \hat{p}_{r|ijkl}$$

for each  $ijkl$ . The traces of the log odds are given in Figure 6. Because the Before/After variable was dropped, pre-OPPE and post-OPPE curves are not available. Also, Atlantic and Pacific Fleet curves would be identical. Even

though the interactions between casualty type and month, ship type, casualty type and month did not appear in the loglinear model of Section 3, these interactions are obvious from the traces of the estimated expected number of CASREPTs.

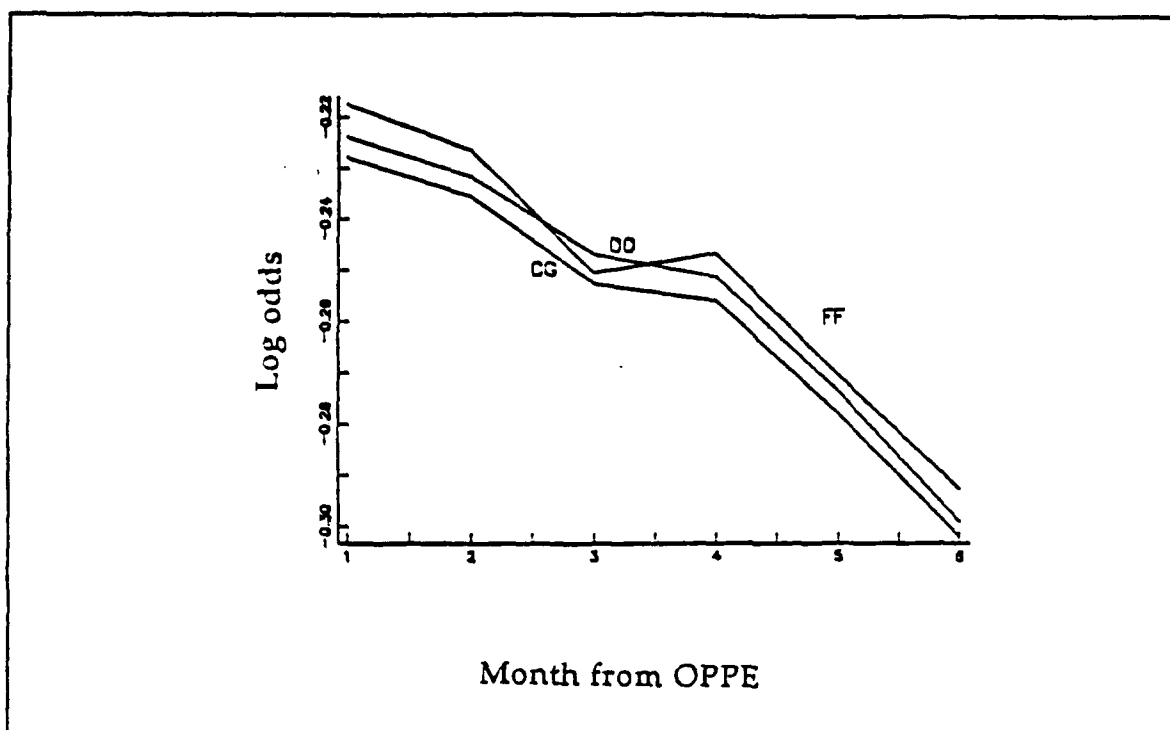


Figure 6. Log Odds vs. Month from OPPE by Ship Type .

## 5. CONCLUSIONS

From the analysis in the previous sections a few things stand out. First, failing to reject a particular model does not mean that it actually fits the data. Different approaches to analyzing the same data can often uncover new relationships. Second, in the analysis in sections 3 and 4 we proceeded as if there was more data than there actually was. In fact, 157 different ships were observed, with only 8 Atlantic Fleet cruisers and 10 Pacific Fleet cruisers.

Thus, although it is clear that CASREPTs tend to increase with proximity to the OPPE exam, that the ratio of engineering to nonengineering CASREPTs tends to decrease with proximity, and that there appears to be a difference between the three ship types, some of the finer distinctions may be due to sampling error. Finally, we did not find one statistical package that could easily handle all aspects of this analysis. All had their drawbacks.

## APPENDIX A. SAS CODE

The following code is an example of the Job Control Language (JCL) and SAS commands that can be used to fit the loglinear model whose parameter estimates are given in Table 12 on MVS at the Naval Postgraduate School. For a detailed explanation of using SAS on MVS see Davis (1990). In this particular example, the data is entered as it appears in Tables 18 and 19, Klingseis (1979). Entering the data in this format necessitates the rather intricate DATA statement. PROC CATMOD is used to fit the loglinear model. Rather than use the POPULATION statement to get maximum likelihood estimates for a product multinomial likelihood, the term SHIP\*CASTYPE\*FLEET\*BA\*MONTH is included and a single multinomial likelihood is maximized. Two files are created, loglin listing sent to the users reader which contains the output from PROCCATMOD and a SAS file PRED.SAS which includes the SAS data set PRED.RESID. Among other things, this data set contains the estimated and observed cell probabilities which can be used to get the standardized residuals. Other SAS PROCs are then used to table and plot these residuals.

```
FILE: EXAMPLE SAS A

//LOGLIN JOB (5096,9999),'L WHITAKER',CLASS=J
// EXEC SAS606,REGION=7000K
//IN1 DD DSN=MSS.F4077.SAS12,USA,DISP=SHR
//RESID DD DISP=(NEW,CATLG,DELETE),UNIT=SYSDA,
//      DCB=(RECFM=FB,LRECL=40,BLKSIZE=23440),SPACE=(23400,(1,1)),
//SYSIN DD *
TITLE 'FINAL MODEL';
DATA RESID.OPPE;
  FORMAT CHARV $5. FLEET $1. SHIP $2. BA $1. NCAS $5.;
  INPUT CHARV $;
  FLEET=SUBSTR(CHARV,1,1);
```

```

SHIP=SUBSTR(CHARV,2,2);
IF LENGTH(CHARV)=4 THEN CASTYPE = 'N';ELSE CASTYPE='E';
BA=SUBSTR(CHARV.LENGTH(CHARV),1);
DO NCAS= 'ZERO', 'ONE', 'TWO', '3PLUS';
  DO MONTH= 1, 2, 3, 4, 5, 6;
    INPUT COUNT @;
    IF COUNT=0 THEN COUNT=1E-20;
    OUTPUT;
  END;
INPUT;
END;

```

CARDS;

PFFEX

```

18 19 20 21 25 28
 7 10  8 14  6  7
 6  6  9  1  4  3
 9  5  3  4  5  2

```

PFFX

```

21 19 19 21 24 27
14 15 13 11  6  9
 2  4  6  3  4  4
 3  2  2  5  6  0

```

PDDEX

```

 8  7 11 10 13 13
 5  7  4  2  1  1
 3  4  3  2  2  1
 2  0  0  4  2  3

```

•  
•  
•

LCGY

```

 2  3  2  3  2  4
 4  1  2  2  2  2
 1  2  1  1  2  0
 1  2  3  2  2  2

```

```

;
PROC CATMOD DATA=RESID.OPPE ORDER=DATA;
  WEIGHT COUNT;
  RESPONSE / OUT=RESID.PRED(
    KEEP=NCAS SHIP FLEET CASTYPE MONTH _PRES_ _OBS_ _RESID_ _TYPE_
    _SEOBS_ _SEPPRED_);
  MODEL NCAS*BA*SHIP*MONTH*FLEET*CASTYPE=_RESPONSE_
    /NODESIGN NOPROFILE NORESPONSE;
  LOGLIN NCAS|SHIP#2
    SHIP*FLEET FLEET
    MONTH*NCAS MONTH
  SHIP*BA*MONTH*FLEET*CASTYPE;
RUN;

```

/\*  
//

## APPENDIX B. MAXIMUM LIKELIHOOD ESTIMATION FOR CENSORED GEOMETRIC AND POISSON DISTRIBUTIONS

The frequency counts  $f_1, \dots, f_4$  represent the number of ships reporting 0, 1, 2, 3 or more casualties, respectively. These are right censored data and the censoring influences the maximum likelihood estimation method. Indeed the estimators developed below (or their equivalents) are necessary to support the *chi-square* test statistics used in goodness-of-fit testing. Both the Geometric and Poisson distributions are candidates to model these frequencies. It is natural to default to the familiar distributions. What follows is an analysis of goodness-of-fit testing when these two distributions are fitted to the frequency counts, pooled over the six months, and treated as 24 separate experiments as in Section 2.

**Geometric.** Consider the censored geometric probability function

$$\begin{aligned} p_j &= qp^j \quad \text{for } j = 0, 1, \dots, c-1, \\ p_c &= p^c \end{aligned} \tag{B.1}$$

and  $p + q = 1$ . The data consists of counts  $f_0, f_1, \dots, f_c$  and let  $N = \sum_{j=0}^c f_j$

We proceed to develop the likelihood function, its logarithm, and the maximum likelihood equations.

$$L(p) = \prod_{j=0}^c p_j^{f_j} = \left\{ \prod_{j=0}^{c-1} [qp^j]^{f_j} \right\} p^{\sum_{j=0}^c f_j}$$

$$\varphi = \ln(L) = \sum_0^{c-1} f_j \ln(q) + \sum_0^{c-1} j f_j \ln(p) = (N - f_c) \ln(q) + S \ln(p)$$

where  $S = \sum_{j=0}^c f_j$ . Then

$$\varphi_p = -\frac{N-f_c}{q} + \frac{S}{p}$$

which is set to zero. The solution for  $p$  is the maximum-likelihood estimator

$$\hat{p} = S / (S + N - f_c) \quad (\text{B.2})$$

Poisson. The censored Poisson probability function is

$$\begin{aligned} p_j &= e^{-\lambda} \lambda^j / j! & \text{for } j = 0, \dots, c-1 \\ p_c &= 1 - \sum_{j=0}^{c-1} p_j \end{aligned} \quad (\text{B.3})$$

Again  $f_0, f_1, \dots, f_c$  are the counting data and  $N = \sum_0^c f_j$ . The general structure of the likelihood system is

$$L(\lambda) = \prod_{j=0}^c p_j^{f_j}$$

$$\varphi = \ln(L) = \sum_0^c f_j \ln(p_j)$$

$$\varphi_\lambda = \sum_0^c \frac{f_j}{p_j} \frac{\partial p_j}{\partial \lambda}.$$

The components of this system are best treated with the following technique.

$$\frac{\partial p_0}{\partial \lambda} = -e^{-\lambda} = -p_0 = p_{-1} - p_0 \quad \text{if } p_{-1} = 0 \text{ by convention.}$$

$$\frac{\partial p_j}{\partial \lambda} = e^{-\lambda} \left[ \frac{\lambda^{j-1}}{(j-1)!} - \frac{\lambda^j}{j!} \right] = p_{j-1} - p_j \quad \text{for } j = 1, \dots, c-1$$

$$\frac{\partial p_c}{\partial \lambda} = - \sum_0^{c-1} \frac{\partial p_j}{\partial \lambda} = \sum_0^{c-1} (p_j - p_{j+1}) = p_{c-1}.$$

Then

$$\frac{1}{p_j} \frac{\partial p_j}{\partial \lambda} = \frac{p_{j-1}}{p_j} - 1 \quad \text{for } j = 0, \dots, c-1$$

$$\frac{1}{p_c} \frac{\partial p_c}{\partial \lambda} = \frac{p_{c-1}}{p_c}$$

These quantities are then placed into spaces giving the structure

$$\begin{aligned}\varphi_\lambda &= \sum_0^{c-1} f_j \left[ \frac{p_{j+1}}{p_j} - 1 \right] + f_c \frac{p_{c-1}}{p_c} \\ &= \sum_0^{c-1} f_j \left[ \frac{j}{\lambda} - 1 \right] + f_c \frac{p_{c-1}}{p_c}.\end{aligned}\tag{B.4}$$

It is required to solve  $\varphi_\lambda = 0$  for  $\lambda$ . This equation is nonlinear and explicit solution is not possible. Newton-Raphson iteration works quite well however. To execute it let  $g(\lambda) = \varphi_\lambda$  and evaluate the derivative

$$g'(\lambda) = - \sum_0^{c-1} j f_j / \lambda^2 + (f_c / p_c) \left[ p_{c-2} p_{c-1} - p_{c-1}^2 / p_c \right]$$

because  $\frac{\partial p_{c-1}}{\partial \lambda} = p_{c-2} p_{c-1}$ , and  $\frac{\partial p_c}{\partial \lambda} = p_{c-1}$ .

The Newton-Raphson iteration formula is

$$\lambda \leftarrow \lambda - g(\lambda) / g'(\lambda)\tag{B.5}$$

and it can be initiated with  $\lambda_0 = \frac{1}{N} \sum_0^c j f_j$ . Stop when  $|g(\lambda)| < \varepsilon$  for some user defined  $\varepsilon > 0$ .

Tables 14 through 17 show the results of fitting the geometric distribution (B.1) to the 24 cases. The estimates for  $p$  and  $-\ln(p)$  are both tabled;  $p$  is the probability of zero CASREPTs and  $-\ln(p)$  can be compared to the  $\lambda$  estimates for the Poisson model. (These are not to be confused with the significance values.) Both Pearson and likelihood ratio *chi-square* test statistics are listed in Table 15. Generally the tests fail, but for different reasons. This accounts

for the large differences in values. Table 16 shows that all but a handful of the tests fail.

Table 17 through 19 show similar results for the Poisson model (B.3). The format is the same and generally so are the results. The two models do not agree with the data, or with each other.

# Fitting the Geometric Distribution

TABLE 14. MAXIMUM LIKELIHOOD ESTIMATES FOR  $p/-\ln(p)$

		ATLANTIC				PACIFIC			
		Engineering		Nonengineering		Engineering		Nonengineering	
FF	Before	.416/	.88	.494/	.71	.479/	.74	.436/	.83
	After	.380/	.97	.486/	.72	.471/	.75	.495/	.70
DD	Before	.451/	.80	.574/	.55	.472/	.75	.487/	.72
	After	.317/	1.15	.492/	.71	.500/	.69	.567/	.57
CG	Before	.364/	1.01	.533/	.63	.539/	.62	.394/	.93
	After	.303/	1.19	.606/	.50	.325/	1.12	.509/	.67

TABLE 15. PEARSON/LIKELIHOOD RATIO CHI-SQUARE (2)  
GOODNESS-OF-FIT VALUES

		ATLANTIC				PACIFIC			
		Engineering		Nonengineering		Engineering		Nonengineering	
FF	Before	4.5/	19.7	9.5/	40.5	15.1/	38.0	5.9/	22.6
	After	1.8/	12.7	9.0/	37.0	6.1/	28.8	4.8/	33.2
DD	Before	31.2/	37.3	18.0/	51.9	6.8/	16.8	20.9/	27.7
	After	10.0/	11.6	6.7/	25.9	2.5/	15.7	21.2/	38.4
CG	Before	9.4/	13.0	14.0/	17.4	8.4/	16.5	4.9/	7.1
	After	1.3/	2.0	15.2/	23.2	3.3/	4.8	3.2/	11.0

TABLE 16. SIGNIFICANCE (1- $p$  VALUES)

		ATLANTIC				PACIFIC			
		Engineering		Nonengineering		Engineering		Nonengineering	
FF	Before	.896/	1.00	.991/	1.00	1.000/	1.00	.947/	1.00
	After	.596/	1.00	.989/	1.00	.952/	1.00	.911/	1.00
DD	Before	1.000/	1.00	1.000/	1.00	.967/	1.00	1.000/	1.00
	After	.993/	1.00	.965/	1.00	.709/	1.00	1.000/	1.00
CG	Before	.991/	1.00	.999/	1.00	.985/	1.00	.915/	.97
	After	.465/	.64	.999/	1.00	.807/	.91	.801/	1.00

# Fitting the Poisson Distribution

TABLE 17. MAXIMUM LIKELIHOOD ESTIMATES FOR  $e^{-\lambda}/\lambda$

		ATLANTIC				PACIFIC			
		Engineering		Nonengineering		Engineering		Nonengineering	
FF	Before	.738/	.478	.937/	.392	.846/	.429	.767/	.464
	After	.665/	.514	.925/	.396	.875/	.417	1.026/	.359
DD	Before	.621/	.537	1.194/	.303	.882/	.414	.748/	.473
	After	.458/	.632	.929/	.395	1.018/	.361	1.093/	.335
CG	Before	.669/	.512	.818/	.441	.964/	.381	.542/	.582
	After	.475/	.622	1.173/	.309	.609/	.544	1.005/	.366

TABLE 18. PEARSON/LIK RATIO CHI-SQUARE (2) GOODNESS-OF-FIT VALUES

		ATLANTIC				PACIFIC			
		Engineering		Nonengineering		Engineering		Nonengineering	
FF	Before	26.5/	26.5	18.3/	18.0	40.7/	38.3	15.7/	15.1
	After	19.4/	19.6	29.0/	29.2	19.2/	19.0	25.6/	25.2
DD	Before	99.9/	56.5	15.8/	15.7	19.8/	20.6	49.2/	35.6
	After	36.5/	26.4	9.3/	9.2	10.4/	10.4	26.7/	26.5
CG	Before	18.0/	22.6	29.1/	19.3	9.2/	7.5	15.9/	9.1
	After	1.1/	1.1	9.9/	8.2	11.3/	13.0	2.1/	2.1

TABLE 19. SIGNIFICANCE (1-p VALUES)

		ATLANTIC				PACIFIC			
		Engineering		Nonengineering		Engineering		Nonengineering	
FF	Before	1.00/	1.00	1.00/	1.00	1.00/	1.00	1.00/	1.00
	After	1.00/	1.00	1.00/	1.00	1.00/	1.00	1.00/	1.00
DD	Before	1.00/	1.00	1.00/	1.00	1.00/	1.00	1.00/	1.00
	After	1.00/	1.00	.99/	.99	.99/	.99	1.00/	1.00
CG	Before	1.00/	1.00	1.00/	1.00	.99/	.98	1.00/	.99
	After	.41/	.43	.99/	.98	1.00/	1.00	.64/	.65

## REFERENCES

- Agresti, A. (1990). *Categorical Data Analysis*, John Wiley and Sons
- Bishop, Y.M.M., Fienberg, S. E., Holland, P. W. (1976). *Discrete Multivariate Analysis: Theory and Practice*, MIT Press.
- Davis, H. (1920), "SAS," Techical Note MVS-08, Naval Postgraduate School, Monterey, CA.
- Fienberg, S. E. (1989). *The Analysis of Cross Classified Categorical Data*, MIT Press.
- Klingseis, F. J. (1979), *An Evaluation of the Impact of the Operational Propulsion Plant Examinations upon Casualty Reports*, Master's thesis, Department of Operations Research, Naval Postgraduate School, Monterey, CA.
- Morrison, A. S., Blach, M. M., Lowe, C. R., MacMahon, B., Yoasa, S. Y. (1973). "Some International Differences in Histology and Survival in Breast Cancer," *International Journal of Cancer*, 11: 261-267.
- SAS, Version 6..06, SAS Institute Inc., SAS Circle, Cary, NC.
- STATGRAPHICS, Version 5, STSC Inc.

## INITIAL DISTRIBUTION LIST

1. Library (Code 52).....2  
     Naval Postgraduate School  
     Monterey, CA 93943-5000
  
2. Defense Technical Information Center .....2  
     Cameron Station  
     Alexandria, VA 22314
  
3. Office of Research Administration (Code 81).....1  
     Naval Postgraduate School  
     Monterey, CA 93943-5000
  
4. Prof. Peter Purdue .....1  
     Code OR-Pd  
     Naval Postgraduate School  
     Monterey, CA 93943-5000
  
5. Prof. Lyn R. Whitaker .....20  
     Code OR-Wh  
     Naval Postgraduate School  
     Monterey, CA 93943-5000
  
6. Prof. Robert R. Read .....20  
     Code OR-Re  
     Naval Postgraduate School  
     Monterey, CA 93943-5000
  
7. Technical Typist .....2  
     Code OR  
     Naval Postgraduate School  
     Monterey, CA 93943-5000
  
8. Center for Naval Analyses .....2  
     4401 Ford Ave.  
     Alexandria, VA 22302-0268